Al-Azhar Bull. Sci. Vol. 24, No. 2 (Dec.): pp. 1-8, 2013.

IDENTICAL BANDS AND STAGGERING IN SUPERDEFORMED ROTATIONAL BANDS FOR ^{193,194,195}TL NUCLEI

A. M. KHALAF [(1)], M.F. ELGABRY [(2)] and N. GAB ALLAH [(3)]

(1) Physics Department, Faculty of Science, Al-AZhar University, Cairo – Egypt
(2) Mathematics & Physics Department, Faculty of Engineering, Al-AZhar University, Cairo – Egypt
(3) Physics Department, Faculty of Science (Girls Branch), Al-AZhar University, Cairo – Egypt

Abstract

A simple collective rotational model has been constructed to investigate the identical bands in odd-odd nucleus ^{194}Tl and its neighbor odd-A nuclei ^{193}Tl and ^{195}Tl, to describe the $\Delta I = 1$ staggering effect in signature partner pairs of odd-A superdeformed bands ^{193}Tl (SD1, SD2) and ^{195}Tl (SD1, SD2) and to describe also the $\Delta I = 2$ staggering observed in ^{194}Tl (SD3).

The model parameters and the bandhead spin ware obtained by adopted best fit method. The systematic variation of the kinematic and dynamic moments of inertia are studied as a function of the rotational frequency, it is found that the blocking effect of the high-j intruder orbital plays an important role. To describe the $\Delta I = 1$ staggering we extracted the differences between the average transitions $I + 2 \rightarrow I \rightarrow I - 2$ energies in one band and the transition $I + 1 \rightarrow I - 1$ energies in its signature partner. To describe $\Delta I = 2$ staggering we calculated the deviation of transition energies from a smooth reference representing the finite difference approximation to the fourth derivative of the transition energies. We noticed transition energies in the nucleus ^{193}Tl is identical to their N + 1, N + 2 neighbors. Also the analysis done allows us to confirm $\Delta I = 1$ staggering in signature partners of ^{193}Tl, ^{195}Tl and $\Delta I = 2$ staggering in SD3 band in ^{194}Tl by performing a staggering parameter analysis.

Introduction

Nuclei are considered to be superdeformed (SD) when the nucleus is very far from spherical shape and acquires an elongated shape that can be represented as an approximate ellipsoid with axes in ratio of approximately 2:1:1. Now more than 350 settled superdeformed rotational bands (SDRB'S) in more then 100 nuclei have been well established in several mass regions of nuclear chart [1, 2].

Spin assignment is one of the most difficult and unsolved problems in the study of nuclear superdeformation. Several theoretical approaches to predict the spin of SDRB'S have been suggested [3-9].

Two of the most interesting phenomena observed in SDRB'S are identical bands (IB'S) and staggering effects. The IB phenomenon [10-12] involves SD bands in different nuclei whose moment of inertia and sometimes even γ-ray transition energies, are nearly identical over a significant fraction of the bands. Some SDRB'S show an unexpected anomalous $\Delta I = 2$ staggering effects in the γ-ray energies (a zigzag behavior as a function of rotational frequency or spin) [13-16]. Two $\Delta I = 4$

rotational sequences are consequently separated into two spin sequences with spin values I + 4n and I + 4n + 2 (n = 1, 2, 3 …) respectively. Several theoretical attempts have been made to understand the ΔI = 2 staggering in SD nuclei [17-20]. There is another kind of staggering in SD odd-A nuclei, the ΔI = 1 staggering in signature partner pairs [21-23].

In this paper a three parameter formula for SDRB'S is suggested. The model is applied to investigate the IB'S and the staggering effects in Tl nuclei.

The model

In our model, the excitation energy E(I) for each state with angular momentum I is given by :

$$E(I) = Eo + a[\sqrt{1 + bI(I+1)} - 1] \tag{1}$$

where E_o is the bandhead energy, a rotational parameter and b characterizes the nuclear softness.

In this expression the rigid rotor limit corresponds to b = 0 and ab keeping finite.

Taking the bandhead energy E_o as constant we can modify the above energy to contain three parameters as the third term describe the effect of anhermenicty:

$$E(I) = a[\sqrt{1 + bI(I+1)} - 1] + cI(I+1) \tag{2}$$

which leads to form the transition energy:

$$\begin{aligned} E_\gamma(I) &= E(I) - E(I-2) \\ &= a[\sqrt{1 + bI(I+1)} - \sqrt{1 + b(I-2)(I-1)}] + 2c(2I+1) \end{aligned} \tag{3}$$

Theoretical Aspects

Spin assignment is one of the most difficult problems in SD nuclei. This is due to the difficulty of establishing the de-excitation of the SD band into known yrast states. Several related fitting procedure for assigning spins have been proposed [3-9].

The nuclear rotational frequency is defined as the first derivative of the energy E(I) with respect to the angular momentum Î,

$$\hbar\omega = \frac{dE(I)}{d\hat{I}} \quad , \quad \hat{I} = \sqrt{I(I+1)} \tag{4}$$

The behavior of moment of inertia in SDRB'S is a strong indicator of their nuclear structure. Two possible types of nuclear moments of inertia have been suggested which reflect two different aspects of nuclear dynamics.

The kinematic moment of inertia $J^{(1)}$ is defined as the inverse of the slope of the curve of energy E(I) versus Î:

$$\frac{J^{(1)}}{\hbar^2} = \hat{I}\left(\frac{dE(I)}{d\hat{I}}\right)^{-1} = \frac{\hat{I}}{\hbar\omega} \tag{5}$$

and the dynamical moment of inertia $J^{(2)}$, which is related to the curvature in the curve E(I) versus Î:

$$\frac{J^{(2)}}{\hbar^2} = \left(\frac{d^2E(I)}{d\hat{I}^2}\right)^{-1} = \frac{1}{\hbar^2}\frac{1}{\omega}\frac{dE(I)}{d\omega} = \frac{1}{\hbar}\frac{d\hat{I}}{d\omega} = J^{(1)} + \omega\frac{dJ^{(1)}}{d\omega} \tag{6}$$

$J^{(1)}$, $J^{(2)}$ in the framework of our proposed model can be written in terms of a, b, c in the form.

$$J^{(1)} = ab[1+bI(I+1)]^{1/2} + \tfrac{1}{2c} \tag{7}$$

$$J^{(2)} = ab[1+bI(I+1)]^{3/2} + \tfrac{1}{2c} \tag{8}$$

The bandhead moments of inertia in terms of a, b, c is given by.

$$J^{(0)} = \frac{\hbar^2}{ab+2c} \tag{9}$$

The experimental quantities of ħω, $J^{(1)}$ and $J^{(2)}$ for SDRB'S are usually extracted from the observed transition energies by using the finite difference approximation:

$$\hbar\omega(I) = \tfrac{1}{4}\left[E_\gamma(I+2\to I) + E_\gamma(I\to I-2)\right] \tag{10}$$

$$J^{(1)}(I) = \frac{2I-1}{E_\gamma(I\to I-2)} \tag{11}$$

$$J^{(2)}(I) = \frac{4}{E_\gamma(I+2\to I) - E_\gamma(I\to I-2)} \tag{12}$$

Therefore, ħω and $J^{(2)}$ does not depend on the knowledge of spin I but depend only on the measured transitions energies, while $J^{(1)}$ depend on spin proposition.

Staggering in SDRB'S

To explore more clearly the $\Delta I = 1$ staggering in signature partner pairs of odd SD bands, one must extract the differences between the average transition $I + 2 \rightarrow I \rightarrow I - 2$ energies in one band and the transition $I + 1 \rightarrow I - 1$ energy in the signature partner:

$$\Delta^{(2)} E_\gamma(I) = \frac{1}{4}[E_\gamma(I+2 \rightarrow I) + E_\gamma(I \rightarrow I-2) - 2E_\gamma(I+1 \rightarrow I-1)] \quad (13)$$

Another remark of the present work is the observation of $\Delta I = 2$ staggering effects in the transition energies for ^{194}Tl (SD3). A few Theoretical proposal for the possible explanation of $\Delta I = 2$ staggering have already been made [19]. The deviation of transition energies from a smooth reference $\Delta^{(4)} E_\gamma(I)$ was determined by calculating the fourth derivative of transition energies at a given spin I resulting to a five-point formula:

$$\Delta^{(4)} E_\gamma(I) = \frac{1}{16}[E_\gamma(I+4) - 4E_\gamma(I+2) + 6E_\gamma(I) - 4E_\gamma(I-2) + E_\gamma(I-4)] \quad (14)$$

Calculated Results and Discussion

The odd-odd nucleus ^{194}Tl and its neighbor odd-A signature partner pairs ^{193}Tl (SD1, SD2) and ^{195}Tl (SD1, SD2) are considered. For each SDRB, the optimized best model parameters a, b, c and the bandhead spin I_o were calculated from the adopted best fit [BFM] [20] of the calculated and experimental transitions energies, the quality of the fit is indicated by the root mean square deviation χ given by:

$$\chi = \left[\frac{1}{N}\sum_i \left(\frac{E_\gamma^{exp}(I_i) - E_\gamma^{cal}(I_i)}{\Delta E_\gamma^{exp}(I_i)}\right)^2\right]^{1/2}$$

where N is the number of the data points entering into the fitting procedure and the $\Delta E_\gamma^{exp}(I_i)$ is the experimental errors in transition energies. The bandhead spin I_o is taken as the nearest half integer. Table (1) gives the best model parameters and the correct bandhead lowest level spin I_o and also the lowest γ-transition energies E_γ ($I + 2 \rightarrow I$) for Tl nuclei.

Using the assigned spin, the rotational frequency $\hbar\omega$, the kinematic $J^{(1)}$ and dynamical $J^{(2)}$ moments of inertia for the above SDRB'S are also obtained. In Figure (1) the calculated $J^{(1)}$ and $J^{(2)}$ are plotted as function of $\hbar\omega$ for the four pairs identical bands (IB'S) = [^{193}Tl (SD1), ^{194}Tl (SD3)], [^{193}Tl (SD1), ^{194}Tl (SD4)], [^{193}Tl (SD1), ^{195}Tl (SD1)] and [^{193}Tl (SD2), ^{195}Tl (SD2)]. The kinematic moment of inertia $J^{(1)}$ is found to be smaller than that of the dynamical moment of inertia $J^{(2)}$, and $J^{(2)}$ increases as $\hbar\omega$ increases. It has been suggested that this rise results from the alignment of angular momentum of paired particles in high intruder orbitals and from the gradual disappearance of pairing correlations with increasing $\hbar\omega$ [3]. A sharp increase in $J^{(2)}$ for ^{195}Tl (SD1) occurs at $\hbar\omega > 0.350$ MeV, while it is not observed for ^{193}Tl (SD1). For the first three sets of IB'S, the blocked proton orbital is 5/2 [642], $\alpha = -1/2$ in frequency range 0.10 0 MeV < $\hbar\omega$ < 0.350 MeV. For the last pair, the blocked proton orbital is 5/2 [642], $\alpha = +1/2$ in frequency range 0.10 0 MeV $< \hbar\omega <$ 0.400 MeV. There are no blocked neutron orbitals in ^{193}Tl. The $J^{(1)}$ and $J^{(2)}$ moments of inertia of our selected SDRB'S can be quantitatively described excellently with the proposed three parameters model. The agreement between the calculated and the experimental transition energies is excellent and the resulting values of spins are very consistent with all spin assignment in other models [1, 2].

To investigate $\Delta I = 1$ staggering in signature partner pairs ^{193}Tl (SD1, SD2) and ^{195}Tl (SD1, SD2), the difference between the average transition I+2 → I, I → I-2 energies in one band and the transition I + 1 → I - 1 energies in its signature partner $\Delta^2 E_\gamma(I)$ are determined and its value as a function of spin I for each signature partner pairs are plotted in Figure (2). The signature partners show large amplitude staggering. $\Delta^2 E_\gamma(I)$ is small at lower spin, increasing faster and faster as the spin I increases.

To explore $\Delta I = 2$ staggering in ^{194}Tl (SD3), the proposed five-point formula is applied as in Figure (3) it is seen that the calculated transition energies for spins I + 4n and I + 4n + 2 (n = 0, 1, 2, 3 …) exhibits staggering behavior. That is the SDRB'S split into two parts with states separated by $\Delta I = 4$, shifting up in energy and the intermediate states shifting down in energy.

Conclusion

The SDRB'S in odd-odd ^{194}Tl nucleus and in signature partners odd-A Tl nuclei have been studied in the framework of simple three parameter collective rotational model. The spin of the observed levels were extracted by assuming various values to the spin of the bandhead at the nearest integer in odd-odd nucleus or half integer in odd-A nuclei. The optimized three parameters have been deduced by using a computer simulated search program in order to obtain a minimum root mean square deviation of the calculated transitions energies from the measured energies. The calculated transition energies, level spins, rotational frequencies, kinematic and dynamic moments of inertia are examined. Four identical bands are found and the $\Delta I = 1$ staggering effects for two pairs in odd-A SDRB'S are investigated. Also the $\Delta I = 2$ staggering in odd-odd ^{194}Tl (SD3) is investigated.

Table (1). The adopted best model parameters a, b, c and the suggested bandhead spin I_o for the selected SDRB'S in $^{193, 194, 195}$Tl.

SDRB'S	a (KeV)	b x10^{-4}	c (KeV)	I_o ($\hbar$)	E_γ (KeV)
^{193}Tl (SD1)	6380.8736	5.3776	3.5196	8.5	206.6
^{193}Tl (SD2)	13573.6591	3.7666	2.6759	9.5	227.3
^{194}Tl (SD3)	12119.9405	1.8708	-0.9722	10	240.5
^{194}Tl (SD4)	22034.6647	2.4110	2.4672	9	220.3
^{195}Tl (SD1)	6380.8738	5.3775	3.5196	5.5	146.2
^{195}Tl (SD2)	33124.3911	2.4266	1.2551	6.5	167.5

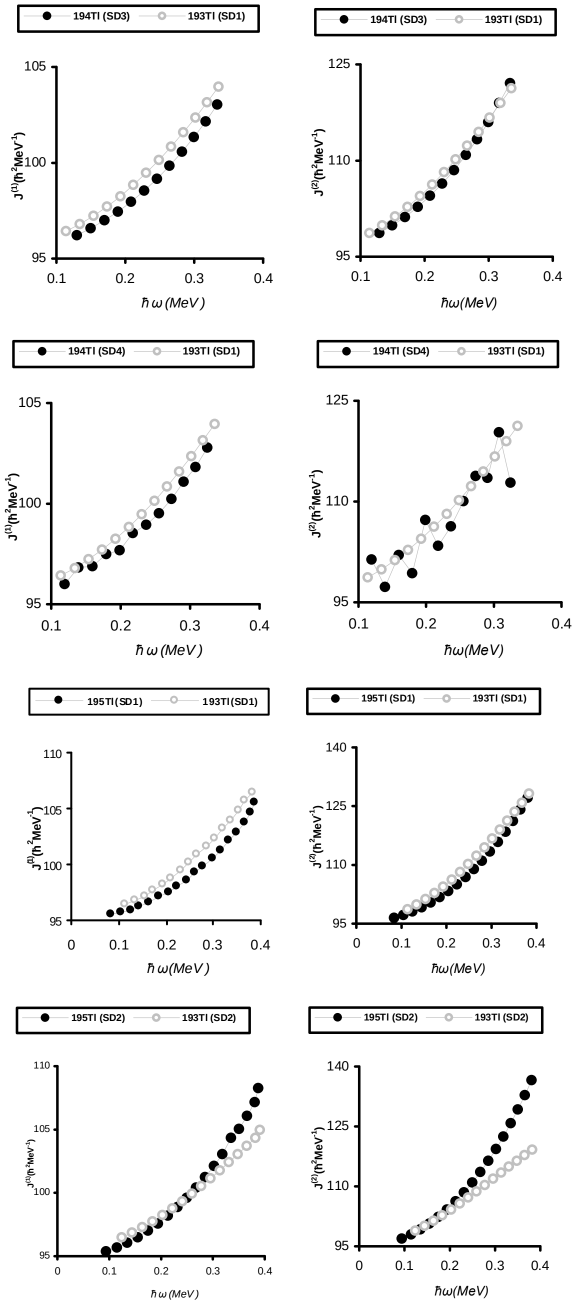

Figure 1. Calculated kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia as a function of rotational frequency ħω for the set of identical bands in ^{193,194,195}Tl nuclei. Open circles for ^{193}Tl and closed circles for both ^{194}Tl and ^{195}Tl.

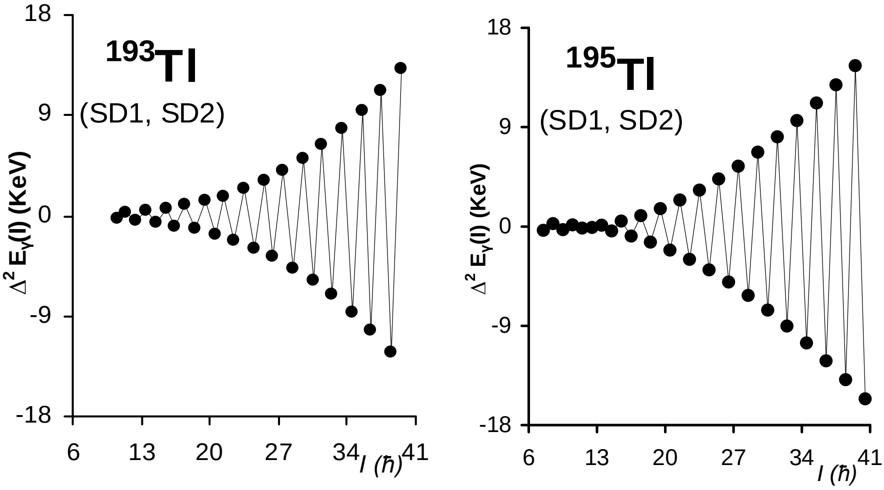

Figure 2. The calculated $\Delta I = 1$ Staggering parameter $\Delta^2 E_\gamma(I)$ extracted from the difference between the average transition $I + 2 \rightarrow I \rightarrow I - 2$ energies in one band and the transition $I + 1 \rightarrow I - 1$ energies in its signature partner plotted as a function of spin I for ^{193}Tl (SD1, SD2) and ^{195}Tl (SD1, SD2).

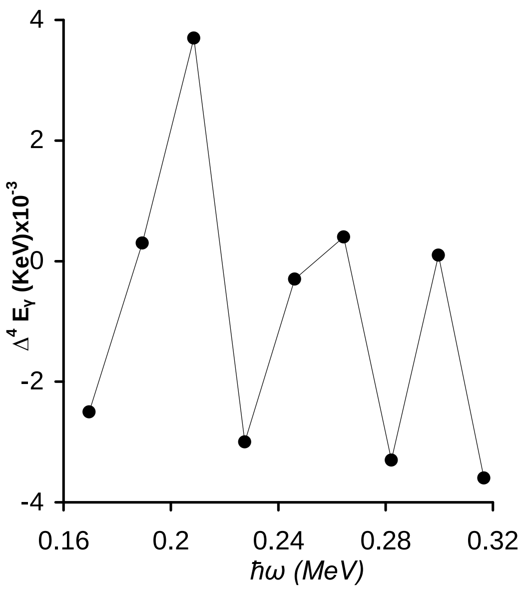

Figure 3. Calculated Staggering parameter $\Delta^4 E_\gamma$ as a function of rotational frequency ħω for ^{194}Tl (SD3).

References

1. B. Singh, Nucl. Data Sheets. 107 (2006) 1.
2. B. Singh, R. Zymvina and R. B. Firestone, Nucl. Data Sheets. 97 (2002) 241.
3. J. A. Becker et al, Nucl. Phys. A520 (1990) 187C.
4. J. Z. Zeng et al, Commun. Theor. Phys. 24 (1995) 125.
5. I. Muntain and A. Sobiczcwski, Phys. Lett. B586 (2004) 254.
6. X. T. He et al, Nucl. Phys. A760 (2005) 263.
7. J. P. Delarache et al, Nucl. Phys. A771 (2006) 103.
8. A. M. Khalaf, M. Allam and E. Saber, Egypt J. Phys. 37. No.3 (2006) 195, 39. No.1 (2008) 41.
9. A. M. Khalaf, M. Allam and M. M. Sirag, Egypt J. Phys. 41. No.2 (2010) 151.
10. C. Baktash, B. Hass and W. Nazarewicz, Ann. Rev. Nucl. Part. Sci. 45 (1995) 485.
11. S. X. Liu, J. Y. Zeng and E. G. Zhao, Phys. Rev. C66 (2002) 024320.
12. A. M. Khalaf, and M. M. Sirag, Egypt J. Phys. 37. No.3 (2006) 277.
13. S. Flibotte et al, Phys. Rev. Lett. 71 (1993) 4299.
14. B. Cederwall et al, Phys. Rev. Lett. 72 (1994) 3150.
15. B. Cederwall et al, Phys. Lett. B346 (1995) 244.
16. D. S. Haslip, Phys. Rev. Lett. 78 (1997) 3447.
17. I. Hamamto and B. R. Mottelson, Phys. Lett. B333 (1995) 294.
18. I. M. Pavlichenkov and S. Flibotte, Phys. Rev. C51 (1995) R460.
19. A. M. Khalaf, and M. M. Sirag, Egypt J. Phys. 35. No.2 (2004) 359.
20. A. M. Khalaf, M. M. Taha and M. Kotob, Progress in Physics 4 (2012) 39.
21. M. P. Carpenter et al, Phys. Rev. C51 (1995) 2400.
22. S. Baunem et al, Phys. Rev. C53 (1996) R9.
23. G. Hackmen et al, Phys. Rev. C55 (1997) 148.