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PROBABILISTIC INVENTORY MODEL WITH LEAD TIME EQUALS SINGLE SCHEDULING PERIOD AND VARYING DETERIORATING COST UNDER CONSTRAINT

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ABSTRACT

In this paper inventory model of declining goods with ambiguous and imprecise details about available storage has been established. Here, our targets are: The optimal scheduling period. The optimal order-level. Minimize the wastage cost due to the deterioration. Minimize the expected average total cost under a restriction on the expected average varying deteriorating cost by using the Lagrange method. This model, is developed for continuously deterioration rate is constant or follows a two-parameter Weibull distribution with varying and constrained expected deteriorating cost, Where the lead time is only one period of time, without shortage and when demand is a random variable during any scheduling time. These probabilistic models are studied in two cases: crisp numbers and trapezoidal fuzzy numbers. Some special cases are deduced. There is a numerical illustration to illustrate the proposed model in the crisp case and the fuzzy case and the sensitivity analysis is performed.

Keywords: Deterioration, Probabilistic demand rate, Scheduling period, Varying cost, Weibull distribution.

1 INTRODUCTION

In many inventory systems the impact of the deterioration is so critical that it cannot be disregarded. By deterioration we mean decay, degradation or spoilage in such a way that the object cannot be used for its original purpose, i.e. the object undergoes a shift in storage such that it loses its value partly or completely over time. In some models the item may be lost over time and therefore its price may decline based on its age, although in some other systems the item will become obsolete due to changes in design or technical advances. In any such cases, the loss of inventory due to deterioration cannot be ignored when evaluating the system, because it provides an incomplete model of inventory systems operations. Photographic videos, medical products and pharmaceuticals, other chemicals, electronic components are some of the examples of items where there may be significant degradation during their usual inventory storage period. Therefore, the failure must be taken into account when deciding on their storage policies. Many probabilistic models have been created for goods that are continually deteriorating in time for example, [13] produced periodic review inventory model for gumbel deteriorating items when demand follows pareto distribution. [12] studied a probabilistic inventory model with two-parameter exponential deteriorating rate and pareto demand distribution. [15] introduced an inventory model for deteriorating items with weibull deterioration with time dependent demand and shortages. [4] presented an inventory model for deteriorating items with quadratic demand and partial backlogging. [3] studied optimal control of production inventory model with exponential deterioration. [6] produced on a probabilistic scheduling period inventory system for deteriorating items with lead time equal to one scheduling period. [5] studied m-scheduling-period inventory model for deteriorating items with instantaneous demand. [2] explained a note on an order-level inventory model for a system with constant rate of deterioration. Many probabilistic inventory models assume that the cost units are constant or that one of these units differs. Hundreds of articles and books present models for this under a wide range of conditions and assumptions. [10] studied probabilistic multi-item inventory model with varying mixture shortage cost under restrictions. [9] deduced probabilistic periodic review $< Q_m, N >$ inventory model using Lagrange technique and fuzzy adapting particle swarm optimization. [14] produced multi-item EOQ model with varying holding cost a geometric programming approach. [11] examined periodic review probabilistic multi-item inventory system with zero lead time under constraints and varying ordering cost. [8] introduce probabilistic single-item inventory problem with varying order cost under two linear constrints. [1] studied probabilistic multi-item inventory model with varying order cost under two restrictions. [7] explained procurement and inventory system: theory and analysis.

In this paper, we use a different approach, in the absence of shortages we find a stochastic stock management mechanism that deals with goods that deteriorate. Radioactive materials used in medical diagnostics, coal or any other form of fuel used for heating boilers-particularly in glass, cement or any other similar industry where heating a furnace up to the certain temperature takes a considerable amount of time and also some medicines used during the treatment of deadly diseases are examples of failing products that could lead to disaster and should therefore not be permitted.

This problem of finding the optimum scheduling period for an inventory model which is subject to continuous decline and stochastic demand is considered here, varying deteriorating costs, a restriction on the expected deteriorating costs, multi-items $(r = 1,2,3,\dots,n)$, non-zero lead time and with lead time equivalent to one scheduling period and then each scheduling period N_r is the lead time for the next scheduling period, that might be the case with certain inventory systems in where the ordered lot comes at the time next order is placed. Under the conditions of no shortage and continuous variable time, the model is built. The mathematical model is analyzed for a deterioration function in a general form, the deterioration rate is constant or follows a two-parameter Weibull distribution and then its particular case is presented. The minimum expected total cost of the system is obtained. We evaluated the optimal policy variables in two sub models: the first is, as usual, when the cost components are considered as crisp values, and the second one is when the cost components are fuzzified as a trapezoidal fuzzy numbers, which is called the fuzzy model. Finally, a numerical example is solved and the sensitivity analysis is conducted to demonstrate the effects of increasing parameter values on the optimal solution for the system.

2 Notations and Assumptions

To develop the inventory model of deteriorating items with varying deteriorating cost, the following notations and assumptions will be used in this paper:

2.1 Notations

n	Number of items.
$MISS$	The multi-item, single source.
t	An element of a random variable represents the time to the deterioration.
X_r	The demand for the r^{th} item.
N_r	The decision variable representing the length of the any scheduling period for the r^{th} item.
$N_r{}^*$	The optimum scheduling period for the r^{th} item.
X_r/N_r	The demand X_r during any scheduling period N_r for the r^{th} item.
$E(X_r/N_r)$	The mean demand X_r during any scheduling period N_r for the r^{th} item.
$\bar{D}_r$	The average demand rate for the r^{th} item.
$\theta_r(t)$	The deteriorating rate of the on hand inventory at time t for the r^{th} item.
θ_r	Constant rate of deterioration for the r^{th} item.
Q_{mr}	The order-level of the system for the r^{th} item.
$Q_{mr} - I_r$	The on hand stock at the time an order is received i.e. at the start of the lead

time period for $r^{\underline{th}}$ item, where I_r is an order place units for the $r^{\underline{th}}$ item.

$Q_{mr}{}^*$	The optimum order-level for the $r^{\underline{th}}$ item.
Q_{1r}	Inventory level initial for the $r^{\underline{th}}$ item for the current period.
$q_{1r}(t)$	The inventory level of the system at various points of time during the lead time for $r^{\underline{th}}$ item.
$q_{2r}(t)$	The inventory location of $r^{\underline{th}}$ item at various points of time during the current period.
$q_{dr}(N_r)$	The number of units required to deteriorate for the $r^{\underline{th}}$ item.
C_{or}	The order cost per unit for the $r^{\underline{th}}$ item.
C_{hr}	The inventory holding cost per unit for the $r^{\underline{th}}$ item.
C_{dr}	The deteriorating cost per unit for the $r^{\underline{th}}$ item.
$C_{dr}(N_r)$	The varying deteriorating cost per unit for the $r^{\underline{th}}$ item, $C_{dr}(N_r) = N_r{}^{\beta} C_{dr}$ (1).
β	A constant real number chose to provide the best fit for estimated expected cost function, $0 < \beta < 1$.
K_{dr}	The limitation on the expected average deteriorating cost for the $r^{\underline{th}}$ item.
λ_{dr}	Lagrange multiplier for the $r^{\underline{th}}$ item.
$\bar{H}_r(N_r)$	The average inventory level per time unit for the $r^{\underline{th}}$ item.
$E(OC_r)$	The expected ordering cost for the $r^{\underline{th}}$ item.
$E(HC_r)$	The expected holding cost for the $r^{\underline{th}}$ item.
$E(DC_r(N_r))$	The expected varying deteriorating cost for the $r^{\underline{th}}$ item.
$E(TC_r(N_r))$	The expected total cost function for the $r^{\underline{th}}$ item.
$E(TC)$	The expected total cost function of the system.
$\min E(TC)$	The minimum expected total cost function.

2.2 Assumptions

1. The demand X_r is a random variable at any scheduling period N_r, for the $r^{\underline{th}}$ item with probability density function (p.d.f.) $f(x_r/N_r)$ and $(x_{r\,min} \le x_r \le x_{r\,max})$.
2. The average demand rate is $\bar{D}_r = \mu_r(N_r)/N_r$ where
$$E(X_r/N_r) = \mu_r(N_r) = \int_{x_{r\,min}}^{x_{r\,max}} x_r \; f(x_r/N_r)dx \qquad (2)$$
is the mean demand during N_r.
3. The replenishment rate is infinite.
4. Not permitted shortages.
5. Non-zero lead time and equal to one scheduling period.
6. The rate of deterioration is constant or follows a Weibull two-parameter distribution, $\theta_r(t) = \eta_r \pi_r t^{\pi_r - 1}$, where $0 < \eta_r << 1$ is the scale parameter, $\pi_r > 1$ is the shape parameter. It is assumed that the deterioration of units increases with time t > 0.
7. The deteriorated inventory is not being repaired or replaced during the time under consideration.

3 The Mathematical Model for Crisp Environmental

Consider the total cost of the system composed of three components. Since the number of replenishments per unit for $r^{\underline{th}}$ item is $1/N_r$, the expected average total cost per unit time of the system during the period is composed of the expected average order cost per unit time of $r^{\underline{th}}$ item, the expected average holding cost per unit time of $r^{\underline{th}}$ item and the expected average varying deteriorating cost of $r^{\underline{th}}$ item as follows:

$$E(TC) = \sum_{r=1}^{n} E(TC_r(N_r)) = \sum_{r=1}^{n} [E(OC_r) + E(HC_r) + E(DC_r(N_r))]$$

$$E(TC) = \sum_{r=1}^{n} [C_{or}/N_r + C_{hr}\,\bar{H}_r(N_r) + C_{dr}(N_r)\, q_{dr}(N_r)/N_r] \tag{3}$$

Notice that every scheduling period N_r is the lead time for the next scheduling period. Then we considered a pair of consequential scheduling periods and we called them respectively "the lead time" and "the current period". Figure 1 show the inventory level of this system.

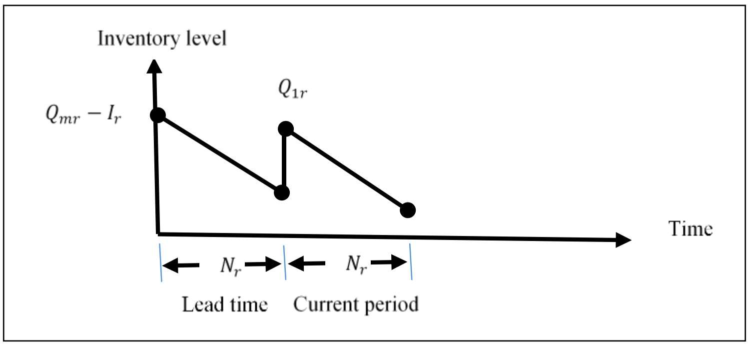

Figure 1: Graphical representation of the inventory level during the lead time period and the current period

Then, note that the degradation in inventory is caused by demands and the deterioration of units, the differential equation representing the inventory level $q_{1r}(t)$ of the system at time $t\ (0 \le t \le N_r)$ during the lead time is demonstrated by the following:

$$\frac{dq_{1r}(t)}{dt} + \theta_r(t) q_{1r}(t) = -\frac{x_r}{N_r} \quad , \qquad 0 \le t \le N_r \tag{4}$$

Using the $q_{1r}(0) = Q_{mr} - I_r$ limit condition, the solution of Eq. (4) is:

$$q_{1r}(t) = \left[Q_{mr} - I_r - \frac{x_r}{N_r} \int_0^t A(t)dt\right] / A(t) \quad , \ (0 \le t \le N_r) \tag{5}$$

Where $$A(t) = e^{\int_0^t \theta_r(t)dt} \quad , \ (0 \le t \le N_r) \tag{6}$$

After the order of I_r' unit has been realized then, the final inventory is:

$$q_{1r}(N_r) + I_r' = \left[Q_{mr} - I_r - \frac{x_r}{N_r} \int_0^{N_r} A(t)dt\right] / A(N_r) + I_r'$$

Which will be independent of I_r, iff $I_r' = I_r/A(N_r)$. For this, an order must be put for $I_r/A(N_r)$ units, So that the final amount of inventory for the lead period after completion of the order is:

$$Q_{1r} = \left[Q_{mr} - \frac{x_r}{N_r} \int_0^{N_r} A(t)dt\right] / A(N_r) \tag{7}$$

Where Q_{1r} will be the inventory level initial for the current period, the inventory would then deplete due to the demand and deteriorate with the preceding consideration. The inventory location $q_{2r}(t)$ of r^{th} item in the interval, $0 \le t \le N_r$ follows the differential equation in the current period with demands x_r is given by:

$$\frac{dq_{2r}(t)}{dt} + \theta_r(t)q_{2r}(t) = -\frac{x_r}{N_r} \quad , \qquad (0 \le t \le N_r) \tag{8}$$

Using the $q_{2r}(0) = Q_{1r}$ limit condition, the solution of Eq. (8) is:

$$q_{2r}(t) = \left[Q_{1r} - \frac{x_r}{N_r}\int_0^t A(t)dt\right] / A(t) \quad , \qquad (0 \le t \le N_r) \tag{9}$$

Some drugs and medicines used in the remedy of deadly diseases are examples of a number of the deteriorating items whose shortage may lead to disastrous results and should not be allowed afterwards. So even the $x_{r\,max}(N_r)$ maximum demand and the degradation of the units during the lead time and also the current period should not allow shortage. That means the Q_{mr} order level has to be from Eqs. (7) and (9):

$$Q_{mr} = \frac{(1 + A(N_r))x_{r\,max}(N_r)}{N_r}\int_0^{N_r} A(t)dt \tag{10}$$

Then, from Eq. (7), (9) and (10), the average inventory level per unit time of the r^{th} item is given by:

$$\begin{aligned}\bar{H}_r(N_r) &= \frac{1}{N_r}\int_0^{N_r} E(q_{2r}(t))\ dt \\ &= \frac{\{(1 + A(N_r))x_{r\,max}(N_r) - N_r\bar{D}_r\}\int_0^{N_r} A(t)dt}{{N_r}^2 A(N_r)}\int_0^{N_r}\frac{dt}{A(t)} \\ &\quad - \frac{\bar{D}_r}{N_r}\int_0^{N_r}\frac{\int_0^t A(t)dt}{A(t)}dt\end{aligned} \tag{11}$$

and the number of units expected to decline for the r^{th} item is given by:

$$\begin{aligned}q_{dr}(N_r) &= E(Q_{1r}) - \mu(N_r) - E\big(q_{2r}(N_r)\big) \\ &= \left\{x_{r\,max} - \frac{(x_{r\,max} - N_r\bar{D}_r)}{(A(N_r))^2}\right\}\frac{\int_0^{N_r} A(t)dt}{N_r} - N_r\bar{D}_r\end{aligned} \tag{12}$$

Then substituting from Eq. (11) and (12) in (3) we shall obtain the expected average total cost per unit time of the system for the period as follows:

$$\begin{aligned}E(TC) = \sum_{r=1}^{n}\Bigg[&\frac{C_{or}}{N_r} - \frac{C_{hr}\bar{D}_r}{N_r}\int_0^{N_r}\frac{\int_0^t A(t)dt}{A(t)}dt - C_{dr}{N_r}^{\beta}\bar{D}_r \\ &+ C_{hr}\frac{\{(1 + A(N_r))x_{r\,max}(N_r) - N_r\bar{D}_r\}\int_0^{N_r} A(t)dt}{{N_r}^2 A(N_r)}\int_0^{N_r}\frac{1}{A(t)}dt \\ &+ C_{dr}{N_r}^{\beta-2}\left\{x_{r\,max} - \frac{(x_{r\,max} - N_r\bar{D}_r)}{(A(N_r))^2}\right\}\int_0^{N_r} A(t)dt\Bigg]\end{aligned} \tag{13}$$

In order to find the optimal N_r scheduling period, it is important to know the implied structure of the $X_{r\,max}$ and $\theta_r(t)$ function.

Consider the following relationship

$$x_{r\,max}(N_r) = a_r\mu_r(N_r) = a_r N_r \bar{D}_r \quad , \quad a_r \geq 1 \tag{14}$$

hence $$x_{r\,min}(N_r) = b_r\mu_r(N_r) = b_r N_r \bar{D}_r \quad , \quad b_r \leq 1$$

where a_r and b_r are known positive constants.

3.1 Constant Deterioration Rate

The model for:

$$\theta_r(t) = \theta_r \rightarrow A(t) = e^{\int_0^t \theta_r(t)dt} = e^{\theta_r t}, (0 \leq t \leq N_r) \tag{15}$$

Therefore substituting from Eq. (14) and (15) in Eq. (13) we get:

$$E(TC) = \sum_{r=1}^{n}\left[\frac{C_{or}}{N_r} + \left(\frac{C_{hr}}{\theta_r} + C_{dr}N_r{}^{\beta}\right)\bar{D}_r\left(\frac{\{a_r - (a_r - 1)e^{-2\theta_r N_r}\}(e^{\theta_r N_r} - 1)}{\theta_r N_r} - 1\right)\right]$$

The objective is to minimize the expected total cost $E\big(TC(N_r)\big)$ under the constraint:

$$C_{dr}N_r{}^{\beta}\bar{D}_r\left(\frac{\{a_r - (a_r - 1)e^{-2\theta_r N_r}\}(e^{\theta_r N_r} - 1)}{\theta_r N_r} - 1\right) - K_{dr} \leq 0$$

To solve this primary function which is a problem of convex programming, let us write the equations of the previews in the form below:

$$E(TC) = \sum_{r=1}^{n}\left[\frac{C_{or}}{N_r} + \left(\frac{C_{hr}}{\theta_r} + C_{dr}N_r{}^{\beta}\right)\bar{D}_r\left(\frac{\{a_r - (a_r - 1)e^{-2\theta_r N_r}\}(e^{\theta_r N_r} - 1)}{\theta_r N_r} - 1\right)\right] \tag{16}$$

Subject to: $$C_{dr}N_r{}^{\beta}\bar{D}_r\left(\frac{\{a_r - (a_r - 1)e^{-2\theta_r N_r}\}(e^{\theta_r N_r} - 1)}{\theta_r N_r} - 1\right) - K_{dr} \leq 0 \tag{17}$$

The Lagrange multipliers technique is used as follows to find the optimal values $N_r{}^*$ for a given $Q_{mr}{}^*$ which minimize (16) under the constraint (17):

$$L(N_r, \lambda_{dr}) = \sum_{r=1}^{n}\left[\frac{C_{or}}{N_r} + \left(\frac{C_{hr}}{\theta_r} + C_{dr}N_r{}^{\beta}\right)\bar{D}_r\left(\frac{\{a_r - (a_r - 1)e^{-2\theta_r N_r}\}(e^{\theta_r N_r} - 1)}{\theta_r N_r} - 1\right) + \lambda_{dr}\left\{C_{dr}N_r{}^{\beta}\bar{D}_r\left(\frac{\{a_r - (a_r - 1)e^{-2\theta_r N_r}\}(e^{\theta_r N_r} - 1)}{\theta_r N_r} - 1\right) - K_{dr}\right\}\right] \tag{18}$$

The optimal value $N_r{}^*$ may be determined by setting each of the corresponding first partial derivatives Eq. (18) equal to zero, which is minimizing $(E(TC))$ the expected total cost.

i.e.

$$\frac{\partial L}{\partial N_r}\Big|_{N_r = N_r{}^*,\ \lambda_{dr} = \lambda_{dr}{}^*} = 0, \quad \frac{\partial L}{\partial \lambda_{dr}}\Big|_{N_r = N_r{}^*,\ \lambda_{dr} = \lambda_{dr}{}^*} = 0$$

then we obtain:

$$(1+\lambda_{dr}{}^{*})N_r{}^{*\beta}C_{dr}\Big[a_r\, g_r-\beta N_r{}^{*}-\ J_r(2N_r{}^{*}+g_r)e^{-2N_r{}^{*}\theta_r}+\ J_r(N_r{}^{*}+g_r)e^{-N_r{}^{*}\theta_r} - a_r\{g_r-N_r{}^{*}\}e^{N_r{}^{*}\theta_r}\Big] = -\frac{C_{hr}}{\theta_r}\left[a_r\left(N_r{}^{*}-\frac{1}{\theta_r}\right)e^{N_r{}^{*}\theta_r}+\ J_r\left(N_r{}^{*}+\frac{1}{\theta_r}\right)e^{-N_r{}^{*}\theta_r}-\ J_r\left(2N_r{}^{*}+\frac{1}{\theta_r}\right)e^{-2N_r{}^{*}\theta_r} + \frac{a_r}{\theta_r}\right]+\frac{C_{or}}{\overline{D}_r} \tag{19}$$

and

$$C_{dr}N_r{}^{*\beta}\overline{D}_r\left[\frac{\left(a_r-J_r\,e^{-2\theta_r N_r{}^{*}}\right)\left(e^{\theta_r N_r{}^{*}}-1\right)}{\theta_r N_r{}^{*}}-\mathbf{1}\right]=K_{dr} \tag{20}$$

where $g_r=(1-\beta)/\theta_r$, $J_r=a_r-1$

Clearly, from (19) and (20) we calculate the optimal scheduling period $N_r{}^{*}$ which are used to determine the minimum ($E(TC)$) expected total cost and substituting from (14), (15) in (10), the optimal order-level is given by:

$$Q_{mr}{}^{*}=\frac{\left(e^{2\theta_r N_r{}^{*}}-\mathbf{1}\right)a_r\overline{D}_r}{\theta_r} \tag{21}$$

3.2 Deterioration Rate Follows Two-Parameter Weibull Distribution

Using $\theta_r(t)=\eta_r\pi_r t^{\pi_r-1}\ \rightarrow\ A(t)=e^{\eta_r t^{\pi_r}}=\sum_{i=0}^{\infty}\frac{(\eta_r t^{\pi_r})^i}{i!}$, $(0\le t\le N_r)$ (22)

As $(0<\eta_r<<1)$, neglecting the $\eta_r{}^2$ and higher powers terms, Then substituting from Eq. (14) and (22) in Eq. (13) we get:

$$E(TC)=\sum_{r=1}^{n}\left[\frac{C_{or}}{N_r}+C_{dr}\overline{D}_r\frac{\{a_r+(a_r-1)(2\pi_r+1)\}\eta_r N_r{}^{\beta+\pi_r}}{\pi_r+1} + C_{hr}\overline{D}_r\left\{\left(2a_r-\frac{3}{2}\right)N_r + \frac{\{\pi_r-(a_r-1)(\pi_r+1)(\pi_r+2)\}\eta_r N_r{}^{\pi_r+1}}{(\pi_r+1)(\pi_r+2)}\right\}\right] \tag{23}$$

To find an optimal values $N_r{}^{*}$and $Q_{mr}{}^{*}$ which minimize $E(TC)$ under the limitations, the Lagrange multiplier technique with the Kuhn-Tacker conditions is used as follows:

$$L(N_r,\lambda_{dr})=\sum_{r=1}^{n}\left[\frac{C_{or}}{N_r}+C_{hr}\overline{D}_r\left\{N_r\left(2a_r-\frac{3}{2}\right)+\frac{\{\pi_r-(a_r-1)(\pi_r+1)(\pi_r+2)\}\eta_r N_r{}^{\pi_r+1}}{(\pi_r+1)(\pi_r+2)}\right\} + C_{dr}\overline{D}_r\frac{\{a_r+(a_r-1)(2\pi_r+1)\}\eta_r N_r{}^{\beta+\pi_r}}{\pi_r+1} + \lambda_{dr}\left\{\left(C_{dr}\overline{D}_r\frac{\{a_r+(a_r-1)(2\pi_r+1)\}\eta_r N_r{}^{\beta+\pi_r}}{\pi_r+1}\right)-K_{dr}\right\}\right] \tag{24}$$

The optimal value N_r^* may be determined by setting each of the corresponding first partial derivatives Eq. (24) equal to zero, which is minimizing the expected total cost. Then the following equations are obtained:

$$\frac{C_{or}}{N_r^{*2}} = C_{hr}\bar{D}_r\left\{2a_r - \frac{3}{2} + \frac{\{\pi_r - (a_r - 1)(\pi_r + 1)(\pi_r + 2)\}\eta_r N_r^{*\pi_r}}{(\pi_r + 2)}\right\}$$

$$+(1+\lambda_{dr}^*)(\beta+\pi_r)\eta_r C_{dr}\bar{D}_r \frac{\{a_r + (a_r - 1)(2\pi_r + 1)\}N_r^{*(\beta+\pi_r-1)}}{\pi_r + 1} \qquad (25)$$

and

$$C_{dr}\bar{D}_r\eta_r \frac{\{a_r + (a_r - 1)(2\pi_r + 1)\}N_r^{*(\beta+\pi_r)}}{\pi_r + 1} = K_{dr} \qquad (26)$$

Clearly, from equations (25) and (26) we calculate the optimal scheduling period N_r^* which are used to determine the minimum $(E(TC))$ expected total cost and the optimal order-level is given by:

$$Q_{mr}^* = a_r\bar{D}_r\left(2N_r^* + \frac{(\pi_r + 3)\eta_r N_r^{*(\pi_r+1)}}{\pi_r + 1}\right) \qquad (27)$$

4 Fuzzy Model and Solution Procedure

In actual inventory systems, the cost parameters and various parameters that include price, marketing, and demand elasticity of providers are in nature imprecise and uncertain. This confusion introduced the Fuzziness notion. Since the model proposed is in a fuzzy environment, a fuzzy decision should be taken to meet the requirements for the decision, and the results should be fuzzy. So that we consider the model in fuzzy environment. Because of uncertainty it is not easy to precisely define all parameters.

Let

$$\tilde{\theta}_r = (\theta_r - \delta_{1r}, \theta_r - \delta_{2r}, \theta_r + \delta_{3r}, \theta_r + \delta_{4r}),$$

$$\tilde{\eta}_r = (\eta_r - \delta_{5r}, \eta_r - \delta_{6r}, \eta_r + \delta_{7r}, \eta_r + \delta_{8r}),$$

$$\tilde{C}_{or} = (C_{or} - \delta_{9r}, C_{or} - \delta_{10r}, C_{or} + \delta_{11r}, C_{or} + \delta_{12r}),$$

$$\tilde{C}_{hr} = (C_{hr} - \delta_{13r}, C_{hr} - \delta_{14r}, C_{hr} + \delta_{15r}, C_{hr} + \delta_{16r})$$

and $$\tilde{C}_{dr} = (C_{dr} - \delta_{17r}, C_{dr} - \delta_{18r}, C_{dr} + \delta_{19r}, C_{dr} + \delta_{20r})$$

be trapezoidal fuzzy numbers, where δ_{ir}, $i = 1,2,\ldots,20$, $r = 1,2,\ldots,n$, are arbitrary positive numbers under the following restrictions:

$$\theta_r > \delta_{1r} > \delta_{2r}, \delta_{3r} < \delta_{4r} \quad , \quad \eta_r > \delta_{5r} > \delta_{6r}, \delta_{7r} < \delta_{8r}$$

$$C_{or} > \delta_{9r} > \delta_{10r}, \delta_{11r} < \delta_{12r} \quad , \quad C_{hr} > \delta_{13r} > \delta_{14r}, \delta_{15r} < \delta_{16r}$$

and $$C_{dr} > \delta_{17r} > \delta_{18r}, \delta_{19r} < \delta_{20r}$$

Hence, the left and right limits α -cuts of $\theta_r, \eta_r, C_{or}, C_{hr}$ and C_{dr} are given as follows:

$$\tilde{\theta}_{r_v}(\alpha) = \theta_r - \delta_{1r} + (\delta_{1r} - \delta_{2r})\alpha \quad , \quad \tilde{\theta}_{r_u}(\alpha) = \theta_r + \delta_{4r} - (\delta_{4r} - \delta_{3r})\alpha,$$

$$\tilde{\eta}_{r_v}(\alpha) = \eta_r - \delta_{5r} + (\delta_{5r} - \delta_{6r})\alpha \quad , \quad \tilde{\eta}_{r_u}(\alpha) = \eta_r + \delta_{8r} - (\delta_{8r} - \delta_{7r})\alpha,$$

$$\tilde{C}_{or_v}(\alpha) = C_{or} - \delta_{9r} + (\delta_{9r} - \delta_{10r})\alpha \quad , \quad \tilde{C}_{or_u}(\alpha) = C_{or} + \delta_{12r} - (\delta_{12r} - \delta_{11r})\alpha,$$

$$\tilde{C}_{hr_v}(\alpha) = C_{hr} - \delta_{13r} + (\delta_{13r} - \delta_{14r})\alpha \quad , \quad \tilde{C}_{hr_u}(\alpha) = C_{hr} + \delta_{16r} - (\delta_{16r} - \delta_{15r})\alpha$$

and

$$\tilde{C}_{dr_v}(\alpha) = C_{dr} - \delta_{17r} + (\delta_{17r} - \delta_{18r})\alpha \ , \quad \tilde{C}_{dr_u}(\alpha) = C_{dr} + \delta_{20r} - (\delta_{20r} - \delta_{19r})\alpha$$

By using signed distance method, the defuzzified value of fuzzy number is given by:

$$\tilde{\theta}_r = \frac{1}{4}[4\theta_r - \delta_{1r} - \delta_{2r} + \delta_{3r} + \delta_{4r}], \qquad \tilde{\eta}_r = \frac{1}{4}[4\eta_r - \delta_{5r} - \delta_{6r} + \delta_{7r} + \delta_{8r}]$$

$$\tilde{C}_{or} = \frac{1}{4}[4C_{or} - \delta_{9r} - \delta_{10r} + \delta_{11r} + \delta_{12r}], \quad \tilde{C}_{hr} = \frac{1}{4}[4C_{hr} - \delta_{13r} - \delta_{14r} + \delta_{15r} + \delta_{16r}]$$

and $\tilde{C}_{dr} = \frac{1}{4}[4C_{dr} - \delta_{17r} - \delta_{18r} + \delta_{19r} + \delta_{20r}]$

Similarly, in the crisp case, the same steps can be applied here, except that the crisp values of $\theta_r, C_{or}, C_{hr}, C_{dr}$ and η_r will be replaced by the fuzzy values of $\tilde{\theta}_r, \tilde{C}_{or}, \tilde{C}_{hr}, \tilde{C}_{dr}$ and $\tilde{\eta}_r$. Then optimal values N_r^*, Q_{mr}^* can be calculated using the same previous equations to minimize expected annual total cost $(E(\widetilde{TC}(N_r)))$ for fuzzy case i.e. equations (16):(21) for constant deterioration and equations (23):(27) for Weibull deterioration.

5 Special Cases

5.1 Put $\beta = 0$ and $K_{dr} \to \infty \Rightarrow C_{dr}(N_r) = C_{dr}$ and $\lambda_{dr} = \mathbf{0}$. Thus Eq. (16) and (19) for the single item is given by:

$$E(TC) = \frac{C_o}{N} + \bar{D}\left(\frac{C_h}{\theta} + C_d\right)\left[\frac{\{a - (a-1)e^{-2\theta N}\}(e^{\theta N} - 1)}{\theta N} - 1\right] \tag{28}$$

and

$$-\frac{C_o}{\bar{\bar{D}}} + \left(\frac{C_h}{\theta} + C_d\right)\left[a\left(N^* - \frac{1}{\theta}\right)e^{N^*\theta} + (a-1)\left(N^* + \frac{1}{\theta}\right)e^{-N^*\theta} - (a-1)\left(2N^* + \frac{1}{\theta}\right)e^{-2N^*\theta} + \frac{a}{\theta}\right] = 0 \tag{29}$$

Eqs. (28) and (29) are the same as those obtained by [7] for the similar model for deteriorating items without any varying or constraint of deteriorating cost for a single item if the lead time is equal to N_r.

5.2 When there is no deterioration, i.e. $\theta_r = 0$ then Eq. (16), (19) and (21) for the single item is given by:

$$E(TC) = [C_o/N + C_h(2a - 3/2)\bar{D}N], \tag{30}$$

$$N^* = \sqrt{2C_o/C_h(4a-3)\bar{D}} \tag{31}$$

and

$$Q_m^* = 2a\bar{D}N^*$$

Eqs. (30) and (31) are the same as those obtained by [7] for the similar model for nondeteriorating items.

6 A Numerical Example

To explain the inventory model described above, take the following parameter values for a hypothetical inventory system as shown in Table 1, Table 2 and $\pi_1 = 2, \pi_2 = 1.5, \pi_3 = 1.8$. During the scheduling period N_r the demand X_r follows the uniform distribution defined by:

$$f(x_r/N_r) = \begin{cases} 1/50N_1 & x_1 \in [0, 50N_1] \\ 1/50N_2 & x_2 \in [0, 50N_2] \\ 1/50N_3 & x_3 \in [50N_3, 100N_3] \\ 0 & \text{otherwise} \end{cases}$$

Table 1: The crisp parameters for multi-item

Parameters	Item 1	*Item 2*	*Item 3*
$C_{or}(per\ order)$	200$	250$	300$
$C_{hr}(per unit/day)$	0.005$	0.0055$	0.0060$
$C_{dr}(per unit)$	5$	6$	7$
θ_r	0.01	0.014	0.018
η_r	0.001	0.005	0.003
K_{dr} (Constant)	25	36	58
K_{dr} (Weibull)	14	27	31

Table 2: The fuzzy parameters for multi-item

Parameters	*Item 1*	*Item 2*	*Item 3*
$\tilde{C}_{or}(per\ order)$	(30,70,210,250)	(40,80,260,300)	(50,90,310,340)
$\tilde{C}_{hr}(per unit/day)$	(0.0045,0.0048,0.0053,0.0055)	(0.0050,0.0051,0.0058,0.0061)	(0.0052,0.0054,0.0062,0.0065)
$\tilde{C}_{dr}(per unit)$	(3,4,6,8)	(3,5,8,10)	(5,6,10,12)
$\tilde{\theta}_r$	(0.004,0.006,0.010,0.014)	(0.006,0.010,0.014,0.018)	(0.010,0.014,0.018,0.022)
$\tilde{\eta}_r$	(0.0005,0.001,0.001,0.0015)	(0.003,0.005,0.006,0.007)	(0.002,0.0025,0.003,0.004)
$\bar{K}_{dr}$ (Constant)	19	28	47
$\bar{K}_{dr}$ (Weibull)	11	22	25

SOLUTION:

We are solve that $(\mu_1,\mu_2,\mu_3) = (25N_1, 25N_2, 75N_3)$, i.e. $(\bar{D}_1,\bar{D}_2,\bar{D}_3) = (25,25,75)$ and if $X_{r\,max}(N_r) = a_r N_r \bar{D}_r$, then $(a_1,a_2,a_3) = (2,2,4/3)$. The optimal value N_r^* can be obtained by solving Eqs. (19) and (20) of the constant deterioration and Eqs. (25) and (26) of the Wiebull deterioration for different values of β and consequently the corresponding optimum order-level Q_{mr}^* and minimum expected total cost of both the crisp and fuzzy environmental for three items are illustrated in Table 3 and Table 4.

Table 3: Crisp and fuzzy values for constant deterioration

	Crisp Case					*Fuzzy Case*			
	β	N_r	λ_{dr}	Q_{mr}	$E(TC_r(N_r))$	$\tilde{N}_r$	$\tilde{\lambda}_{dr}$	$\tilde{Q}_{mr}$	$E(\widetilde{TC}_r(\tilde{N}_r))$
Item 1	0.1	6.75257	0.022467	722.979	56.6837	5.80520	0.08448	610.131	44.9108
	0.2	5.74426	0.119338	608.723	61.5797	5.00491	0.17283	522.400	48.5229
	0.3	5.01275	0.195265	527.264	66.4397	4.41641	0.23991	458.643	52.0701
	0.4	4.46209	0.254428	466.725	71.1965	3.96843	0.29030	410.535	55.5112
	0.5	4.03503	0.300007	420.232	75.8104	3.61773	0.32745	373.129	58.8232
	0.6	3.69565	0.334506	383.566	80.2588	3.33677	0.35400	343.322	61.9950
	0.7	3.42040	0.359911	354.010	84.5300	3.10729	0.37205	319.083	65.0228
	0.8	3.19322	0.377909	329.739	88.6202	2.91377	0.38324	299.029	67.9069
	0.9	3.00297	0.389765	309.499	92.5299	2.75634	0.38889	282.194	70.6510
Item	0.1	5.89359	0.045779	640.776	80.3930	4.98563	0.07163	529.616	63.7702

	0.2	5.06914	0.123033	544.653	87.0218	4.35241	0.13492	458.785	68.5221
	0.3	4.46532	0.182107	475.647	93.4917	3.88159	0.18102	406.814	73.1037
	0.4	4.00705	0.226655	424.048	99.7430	3.51978	0.21373	367.272	77.4854
	0.5	3.64912	0.259516	384.260	105.744	3.23421	0.23592	336.305	81.6549
	0.6	3.36289	0.282941	352.630	111.479	3.00380	0.24981	311.472	85.6100
	0.7	3.12945	0.298729	327.066	116.947	2.81442	0.25714	291.164	89.3550
	0.8	2.93587	0.308331	305.992	122.149	2.65630	0.25928	274.279	92.8976
	0.9	2.77303	0.312917	288.353	127.095	2.52249	0.25731	260.041	96.2477
Item 3	0.1	4.56632	0.000975	992.617	126.071	3.60644	0.03060	764.556	103.588
	0.2	4.01986	0.049558	865.056	134.721	3.23893	0.05778	682.546	109.617
	0.3	3.60955	0.083798	770.912	142.992	2.95772	0.07329	620.442	115.273
	0.4	3.29176	0.106911	698.948	150.852	2.73641	0.08012	571.958	120.561
	0.5	3.03932	0.121352	642.366	158.290	2.55819	0.08047	533.163	125.500
	0.6	2.83454	0.129016	596.843	165.315	2.41189	0.07601	501.481	130.109
	0.7	2.66546	0.131379	559.506	171.942	2.28982	0.06795	475.161	134.412
	0.8	2.52372	0.129595	528.384	178.189	2.18656	0.05724	452.976	138.434
	0.9	2.40337	0.124573	502.018	184.079	2.09816	0.04459	434.040	142.195

Table 4: Crisp and fuzzy values for Weibull deterioration

	Crisp Case					***Fuzzy Case***			
	β	N_r	λ_{dr}	Q_{mr}	$E(TC_r(N_r))$	$\tilde{N}_r$	$\tilde{\lambda}_{dr}$	$\tilde{Q}_{mr}$	$E(\widetilde{TC}_r(\tilde{N}_r))$
Item 1	0.1	6.31817	0.01222	652.835	47.6029	5.503348	0.02870	564.225	38.150
	0.2	5.81033	0.06062	597.379	50.2168	5.09286	0.07155	520.294	40.0751
	0.3	5.38238	0.10326	551.232	52.8240	4.74487	0.10865	483.389	41.9845
	0.4	5.01784	0.14075	512.313	55.4127	4.44681	0.14069	452.009	43.8706
	0.5	4.70432	0.17362	479.107	57.9734	4.18916	0.16826	425.042	45.7276
	0.6	4.43232	0.20234	450.488	60.4991	3.96459	0.19185	401.652	47.5512
	0.7	4.19452	0.22734	425.602	62.9844	3.76741	0.21193	381.197	49.3384
	0.8	3.98514	0.24900	403.788	65.4252	3.59311	0.22888	363.176	51.0871
	0.9	3.79960	0.26766	384.532	67.8187	3.43808	0.24305	347.195	52.7959
Item 2	0.1	5.43322	0.02416	574.286	74.8417	4.41062	0.05387	460.366	62.0269
	0.2	4.91834	0.07224	515.975	79.4903	4.04192	0.08894	419.711	65.4215
	0.3	4.50178	0.11209	469.527	84.0566	3.74014	0.11653	386.797	68.7154
	0.4	4.15906	0.14484	431.781	88.5193	3.48928	0.13784	359.674	71.9000
	0.5	3.87298	0.17149	400.582	92.8642	3.27793	0.15387	336.985	74.9711
	0.6	3.63114	0.19288	374.421	97.0827	3.09775	0.16547	317.756	77.9275
	0.7	3.42442	0.20975	352.207	101.170	2.94257	0.17332	301.275	80.7698
	0.8	3.24597	0.22270	333.139	105.124	2.80768	0.17802	287.009	83.5003
	0.9	3.09057	0.23230	316.613	108.944	2.68947	0.18007	274.552	86.1220
Item 3	0.1	4.73933	0.03328	987.972	96.7710	3.96935	0.00548	817.267	76.7695
	0.2	4.38461	0.06707	909.178	101.709	3.70496	0.02882	760.282	80.1872
	0.3	4.08661	0.09519	843.808	106.545	3.48096	0.04728	712.390	83.5047
	0.4	3.83331	0.11841	788.803	111.264	3.28909	0.06158	671.640	86.7176
	0.5	3.61577	0.13737	741.954	115.860	3.12316	0.07234	636.589	89.824
	0.6	3.42722	0.15262	701.627	120.327	2.97842	0.08007	606.153	92.8239
	0.7	3.26246	0.16464	666.588	124.662	2.85120	0.08523	579.503	95.7185
	0.8	3.11740	0.17385	635.892	128.865	2.73858	0.08818	555.992	985099
	0.9	2.98885	0.18061	608.801	132.937	2.63828	0.08923	535.111	101.201

7 Sensitivity Analysis

A sensitivity analysis is performed to verify the stability of the model according to different values of parameters θ_r and η_r. We trade one parameter at a time retaining the opposite unchanged parameters, find $N_r^{*'}$, $Q_{mr}^{*'}$, $\min E_r(TC(N_r))'$, then calculate the following sensitivity measure $S(N_r) = \left[\left(N_r^{*'}/N_r^{*}\right) - 1\right] \times 100$, similarly for Q_{mr} and $E_r(TC(N_r))$. The results are summarized in Table 4 and Table 5.

Moreover, we know that the optimal values of N_r would have been N_{r0}^{*} for three items from Eq. (31) was disregarded degradation. Then find $\min E_r(TC(N_{r0}^{*}))'$ for the three items that would have been in this case with each change in θ_r and η_r. Finally, measure the potential cost reductions using the next formula as a compare model for nondeteriorating products:

$$PCR_r = [1 - \{\min E_r(TC(N_r))'/\min E_r(TC(N_{r0}^{*}))'\}] \times 100$$

The results of the sensitivity analysis mentioned above and the potential saving of both the crisp and fuzzy environmental for the three items are illustrated in Table 5: Table 8.

Table 5: Sensitivity analysis and potential savings for the three item with respect to θ_r

Crisp	θ_r	$N_r^{*'}$	$Q_{mr}^{*'}$	$\min E(TC_r(N_r))'$	$S(N_r)$	$S(Q_{mr})$	$S(E(TC_r))$	$\min E(TC_r(N_{r0}))'$	PCR_r
Item 1	0.006	10.73	1145.6	46.9181	58.967	58.461	-17.228	78.0760	39.907
	0.008	8.268	883.99	51.7194	22.443	22.271	-8.7579	97.7843	47.109
	0.010	6.753	722.98	56.6837	0.0000	0.0000	0.0000	117.099	51.593
	0.012	5.723	613.47	61.6965	-15.247	-15.147	8.8436	136.090	54.665
	0.014	4.976	533.93	66.7139	-26.310	-26.148	17.695	154.825	56.910
Item 2	0.010	7.997	867.20	69.9423	35.690	35.336	-12.999	148.493	52.899
	0.012	6.778	736.05	75.1549	15.007	14.868	-6.5156	172.816	56.512
	0.014	5.894	640.78	80.3930	0.0000	0.0000	0.0000	196.822	59.155
	0.016	5.221	568.29	85.6289	-11.407	-11.313	6.5129	220.601	61.184
	0.018	4.692	511.19	90.8495	-20.383	-20.223	13.007	244.238	62.803
Item 3	0.014	5.737	1244.8	113.271	25.644	25.403	-10.153	299.426	62.171
	0.016	5.082	1103.7	119.673	11.292	11.191	-5.0748	338.642	64.661
	0.018	4.566	992.62	126.071	0.0000	0.0000	0.0000	378.150	66.661
	0.020	4.150	902.76	132.453	-9.1267	-9.0522	5.0619	418.035	68.315
	0.022	3.805	828.51	138.812	-16.663	-16.532	10.106	458.383	69.717

Table 6: Sensitivity analysis and potential of fuzzy values savings for the three items with respect to $\tilde{\theta}_r$

Fuzzy	$\tilde{\theta}_r$	$\tilde{N}_r^{*'}$	$\tilde{Q}_{mr}^{*'}$	$\min E(\widetilde{TC}_r(N_r^{*}))'$	$S(\tilde{N}_r)$	$S(\tilde{Q}_{mr})$	$S(E(\widetilde{TC}_r))$	$\min E(TC_r(\tilde{N}_{r0}^{*}))'$	$\widetilde{PCR}_r$
Item 1	(0.000,0.002,0.006,0.010)	10.341	1083.8	35.7375	78.136	77.628	-20.4255	54.1261	33.974
	(0.002,0.004,0.008,0.012)	7.4060	777.42	40.1937	27.575	27.419	-10.5031	71.5984	43.862
	(0.004,0.006,0.010,0.014)	5.8052	610.13	44.9108	00.000	00.000	00.0000	88.7164	49.377
	(0.006,0.008,0.012,0.016)	4.7919	504.13	49.6967	-17.455	-17.373	10.6565	105.5260	52.906
	(0.008,0.010,0.014,0.018)	4.0905	430.69	54.4897	-29.5381	-29.410	21.3289	122.072	55.363
Item 2	(0.002,0.006,0.010,0.014)	7.2034	763.49	54.0177	44.4829	44.1591	-15.2932	108.677	50.295
	(0.004,0.008,0.012,0.016)	5.8828	624.28	58.8716	17.9946	17.8743	-7.68158	130.784	54.985
	(0.006,0.010,0.014,0.018)	4.9856	529.62	63.7702	00.0000	00.0000	0.00000	152.526	58.191
	(0.008,0.012,0.016,0.020)	4.3347	460.88	68.6715	-13.0553	-12.9791	7.68596	173.963	60.525
	(0.010,0.014,0.018,0.022)	3.8400	408.59	73.5579	-22.9781	-22.8513	15.3484	195.153	62.308

Item 3	(0.006,0.010,0.014,0.018)	4.6837	991.42	91.5373	29.8705	29.6723	-11.6329	244.532	62.566
	(0.008,0.012,0.016,0.020)	4.0716	862.56	97.5665	12.8982	12.8177	-5.8125	281.562	65.348
	(0.010,0.014,0.018,0.022)	3.6064	764.56	103.588	00.0000	00.0000	00.0000	318.658	67.493
	(0.012,0.016,0.020,0.024)	3.2405	687.41	109.587	-10.1482	-10.0909	5.79209	355.880	69.207
	(0.014,0.018,0.022,0.026)	2.9447	625.02	115.560	-18.3501	-18.2510	11.5579	393.287	70.617

Table 7: Sensitivity analysis and potential savings for the three items with respect to η_r

Crisp	η_r	$N_r^{*'}$	$Q_{mr}^{*'}$	$\min E(TC_r(N_r))'$	$S(N_r)$	$S(Q_{mr})$	$S(E(TC_r))$	$\min E(TC_r(N_{r0}))'$	PCR_r
Item 1	0.0006	8.0581	831.97	41.3052	27.5388	27.4401	-13.2297	169.513	75.633
	0.0008	7.0265	725.78	44.6306	11.2110	11.1731	-6.24403	220.746	79.782
	0.0010	6.3182	652.84	47.6029	00.0000	00.0000	0.00000	271.980	82.498
	0.0012	5.7928	598.71	50.3118	-8.31577	-8.29003	5.69057	323.214	84.434
	0.0014	5.3828	556.47	52.8149	-14.8048	-14.7605	10.9489	374.447	85.895
Item 2	0.0030	7.4767	788.95	62.9549	37.6117	37.3786	-15.8826	227.522	72.330
	0.0040	6.2463	659.74	69.1262	14.9658	14.8800	-7.63687	297.182	76.740
	0.0050	5.4332	574.29	74.8417	00.0000	00.0000	0.00000	366.843	79.598
	0.0060	4.8481	512.75	80.1981	-10.7698	-10.7147	7.1569	436.503	81.627
	0.0070	4.4028	465.90	85.2633	-18.9655	-18.8726	13.9248	506.163	83.155
Item 3	0.0010	8.4495	1757.4	70.9118	78.2857	77.8791	-26.7220	248.176	71.427
	0.0020	5.8667	1222.0	85.1949	23.7883	23.6822	-11.9624	471.251	81.922
	0.0030	4.7393	987.97	96.7710	00.0000	0.0000	0.0000	694.327	86.063
	0.0040	4.0734	849.68	106.772	-14.0506	-13.9974	10.3343	917.403	88.362
	0.0050	3.6221	755.90	115.714	-23.5746	-23.4901	19.5752	1140.48	89.854

Table 8: Sensitivity analysis and potential of fuzzy values savings for the three items with respect to $\tilde{\eta}_r$

Fuzzy	$\tilde{\eta}_r$	$\tilde{N}_r^{*'}$	$\tilde{Q}_{mr}^{*'}$	$\min E(\widetilde{TC}_r(N_r))'$	$S(\tilde{N}_r)$	$S(\tilde{Q}_{mr})$	$S(E(\widetilde{TC}_r))$	$\min E(\widetilde{TC}_r(N_{r0}))'$	$\widetilde{PCR}_r$
Item 1	(0.0001,0.0006,0.0006,0.0006)	7.0189	719.18	33.1288	27.5388	27.4633	-13.1618	123.791	73.238
	(0.0003,0.0008,0.0008,0.0013)	6.1203	627.32	35.7776	11.2110	11.1820	-6.21873	160.634	77.727
	(0.0005,0.0010,0.0010,0.0015)	5.5033	564.23	38.1500	00.0000	00.0000	0.00000	197.478	80.681
	(0.0007,0.0012,0.0012,0.0017)	5.0457	517.42	40.3149	-8.31577	-8.29609	5.67472	234.321	82.795
	(0.0009,0.0014,0.0014,0.0019)	4.6886	480.88	42.3171	-14.8048	-14.7709	10.9230	271.164	84.394
Item 2	(0.001,0.003,0.004,0.005)	5.9521	620.49	52.5643	34.9499	34.7828	-15.2557	196.442	73.242
	(0.002,0.004,0.005,0.006)	5.0333	525.07	57.4682	14.1186	14.0558	-7.34957	252.194	77.213
	(0.003,0.005,0.006,0.007)	4.4106	460.37	62.0269	00.0000	00.0000	0.00000	307.946	79.858
	(0.004,0.006,0.007,0.008)	3.9552	413.03	66.3110	-10.3243	-10.2831	6.90681	363.699	81.798
	(0.005,0.007,0.008,0.009)	3.6049	376.58	70.3708	-18.2687	-18.1989	13.4520	419.451	83.223
Item 3	(0.000,0.0005,0.001,0.002)	7.4239	1525.9	55.3700	87.0296	86.7047	-27.8750	179.241	69.109
	(0.001,0.0015,0.002,0.003)	4.9708	1022.8	67.2539	25.2286	25.1488	-12.3951	361.154	81.378
	(0.002,0.0025,0.003,0.004)	3.9694	817.27	76.7695	00.0000	00.0000	00.0000	543.068	85.864
	(0.003,0.0035,0.004,0.005)	3.3923	698.77	84.9410	-14.5383	-14.4995	10.6441	724.982	88.284
	(0.004,0.0045,0.005,0.006)	3.0062	619.46	92.2224	-24.2650	-24.2039	20.1289	906.895	89.831

8 CONCLUSION

From the above described example and sensitivity analysis we found:

- The optimal values of N_r , Q_{mr} decreases when β is increase of the constant deterioration and the Weibull deterioration for the three items as shown in Table 3 and Table 4 respectively of both the crisp and fuzzy environmental.
- The minimum expected total cost decrease when β is decrease of the constant deterioration and the Weibull deterioration for the three items as shown in Table 3 and Table 4 respectively of both the crisp and fuzzy environmental.
- The optimal values of N_r and Q_{mr} decrease when θ_r and η_r are increases for the three items in both the crisp and fuzzy cases as shown in Table 5: Table 8, respectively.
- The minimum expected total cost and the potential savings decrease when θ_r and η_r decrease for the three items in both the crisp and fuzzy cases as shown in Table 5: Table 8, respectively.

In other words, if $\beta = 0.1$, it gives the best value for the minimum expected total cost, we can conclude that the minimum expected total cost in fuzzy case is less than in the crisp case, which indicates that the fuzziness is very close to the actuality of life and gets minimum expected total cost less than the crisp case. The optimal values of N_r, $E(TC_r)$ and Q_{mr} are all sensitive with respect to θ_r and η_r and significant potential cost reductions are often present as compared to an analogous nondeteriorating model. This means that the parameter θ_r plays an important role in the assumed inventory system in the sense that a small change in it can cause significant disruptions in optimal system decisions and should therefore be precisely controlled.

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نموذج المخزون الاحتمالي بفترة سماح مساوية لفترة جدولة واحدة مع تغير تكلفة التدهورة في ظل القيود

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الملخص العربي

في هذا البحث تم وضع نموذج جرد للسلع المتدهورة مع تفاصيل غامضة وغير دقيقة حول التخزين المتاح.

أهداف البحث: 1- فترة الجدولة المثلى 2- مستوى المخزون الأمثل 3- تقليل التكلفة المتوقعة بسبب التدهور 4- تقليل متوسط التكلفة الإجمالية المتوقعة في ظل القيد باستخدام طريقة لاغرانج. تم تطوير هذا النموذج من أجل أن يكون معدل التدهور المستمر ثابتًا أو يتبع توزيع وايبل ذي المعلمتين بتكلفة متغيرة ومقيدة للتدهور المتوقع ، حيث يكون زمن التسليم مساويا لفترة زمنية واحدة فقط، عدم السماح بالعجز وعندما يكون الطلب متغيرًا عشوائيًا خلال أي جدولة زمنية. تمت دراسة هذه النماذج الاحتمالية في حالتين: الأعداد المحددة والفازية.